Chapter 4
Electrical Principles

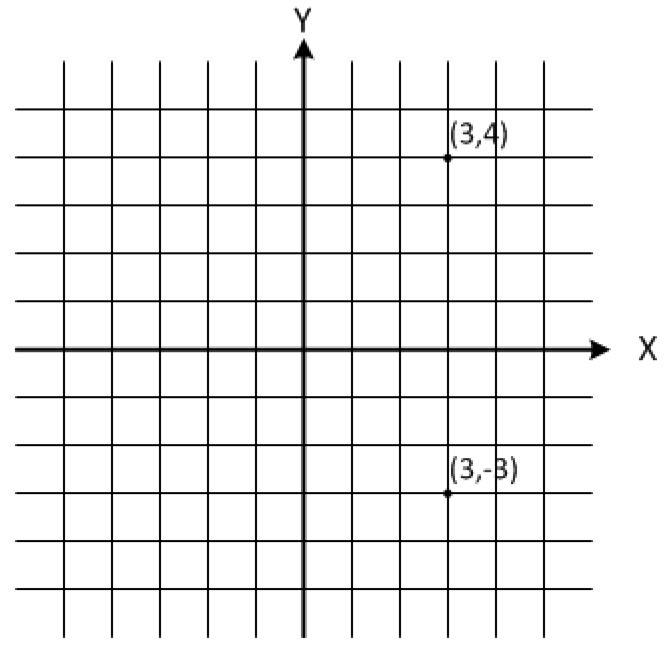
Radio Mathematics

Volts, Amps, Resistance / Impedance

- Can’t see
- Use equations to describe
- Graphs are pictures of equations
- Two types of graphs that show the same data in two different ways

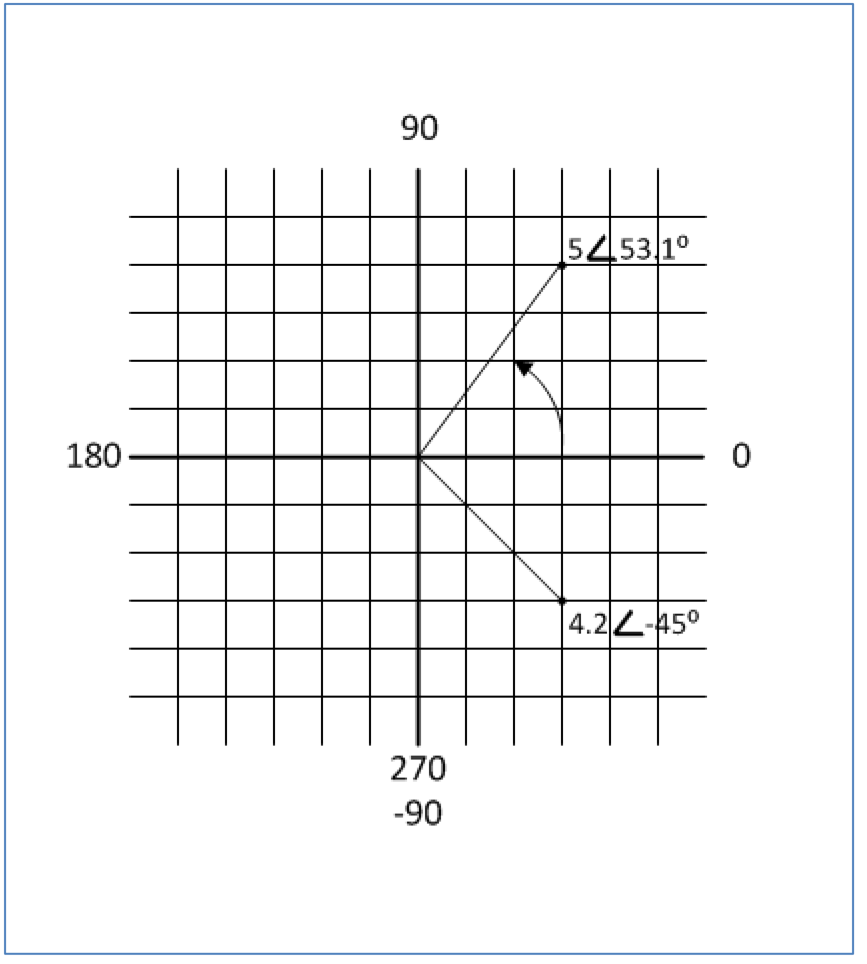
Rectangular Coordinates

- Graph is composed of two axis, horizontal (x) and vertical (y)
- The point (0,0) is called the origin
- Data point is identified as a (x,y) data pair



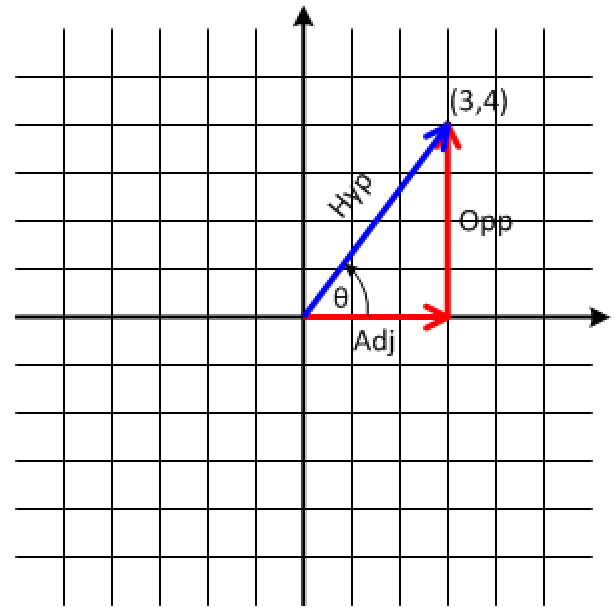
Polar Coordinates

- Graph is composed of a fixed point called a pole (analogous to origin)
- Data point (called a ray) is identified by a distance from the pole and an angle from the horizontal axis (counter clock wise is positive)
- Angles are measured in degrees and radians
  - One cycle / circle = 360 degrees = 2 π radians (1 radian ≈ 57.3°)



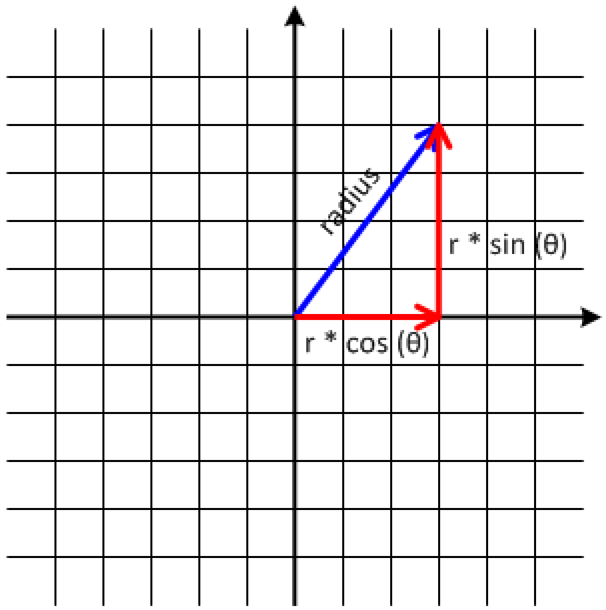
Convert from Rectangular to Polar

- Hyp = $\sqrt{(\text{Adj})^2 + (\text{Opp})^2}$
- $\theta = \tan^{-1}$ (Opp / Adj)



Convert from Polar to Rectangular

- Radius (r) = Hyp
- Adj = r * cos θ
- Opp = r * sin θ



Complex numbers / coordinates

Imaginary numbers

- $\sqrt{36}$ = ?

  $\sqrt{36}$ = 6

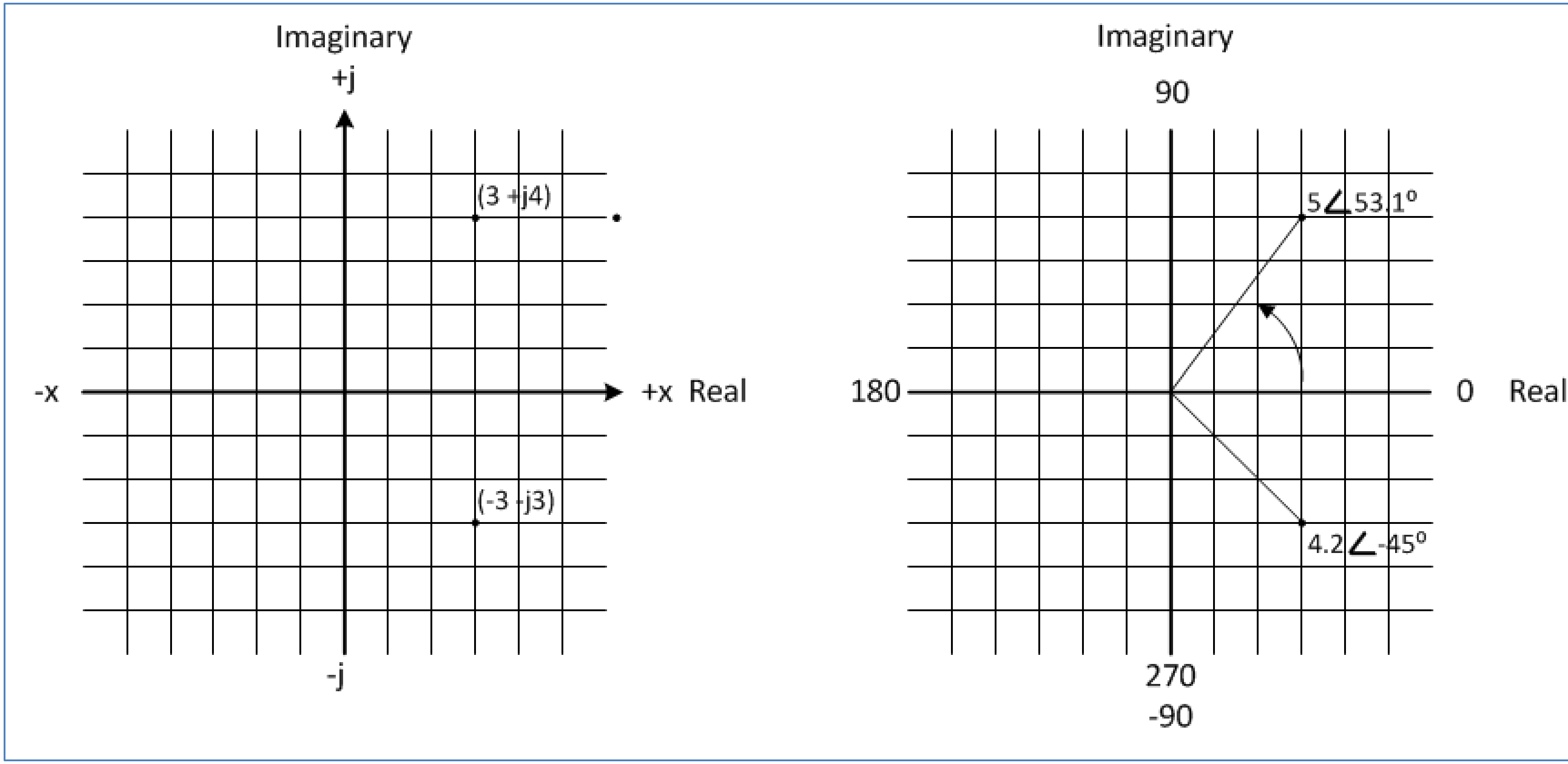
  $\sqrt{(4 * 9)}$ = $\sqrt{4}$ * $\sqrt{9}$ = 2 * 3 = 6

- $\sqrt{-25}$ = ?

  $\sqrt{(25 * -1)}$ =$\sqrt{25}$ * $\sqrt{-1}$ = 5 * $\sqrt{-1}$

- $\sqrt{-1}$ is defined as “imaginary number“
- Mathematicians use ***i***, Electrical Engineers use ***j*** because “***i***" is used to indicate instantaneous current

Complex Coordinates



- Combine real & imaginary numbers
  (real + j imaginary)

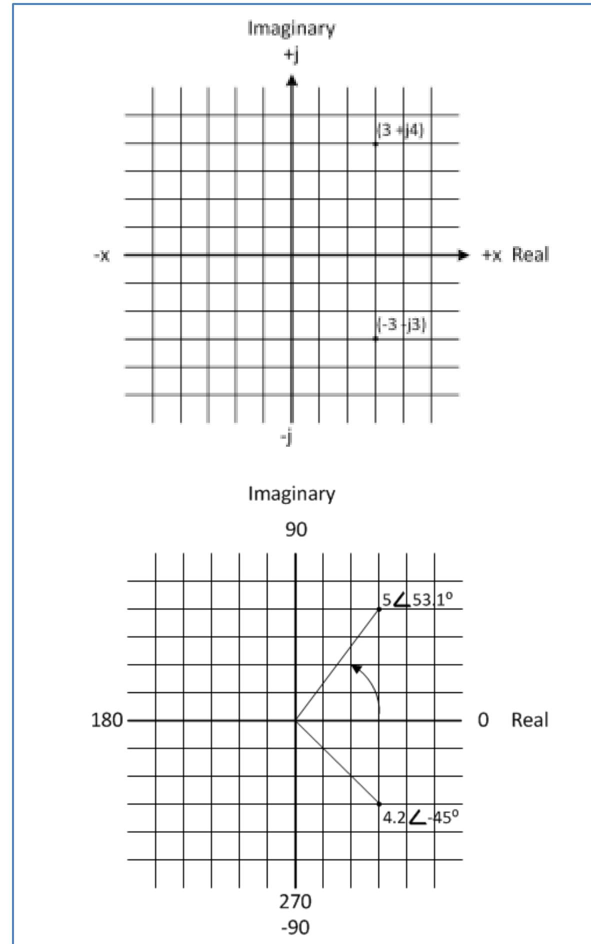
Convert Rectangular <> Polar

Rectangular to Polar

- Hyp = $\sqrt{(\text{Adj})^2 + (\text{Opp})^2}$
- 5 = $\sqrt{(3)^2 + (4)^2}$
- θ = tan $^{-1}$ (Opp / Adj)
- 53.1^{o} = tan $^{-1}$ (4 / 3)

Polar to Rectangular

- Adj = r * cos θ
- 3 = 4.2 * cos(-45^{o})
- Opp = r * sin θ
- -3 = 4.2 * sin (-45^{o})



Complex numbers Math

Add/Subtract - Use Rectangular form

- $(a + j\,b) + (c + j\,d) = (a + c) + j\,(b + d)$
- $(2 + j3) + (4 + j1) = (6 + j4)$

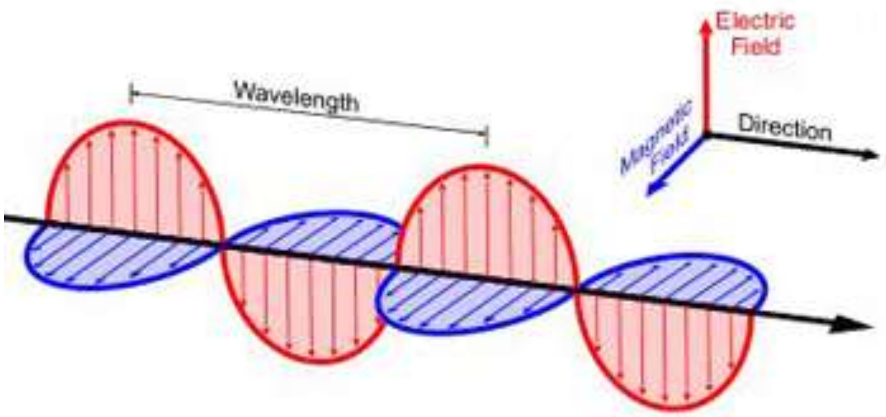
Multiply/Divide - Use Polar form

- $a\angle\theta_1 * b\angle\theta_2 = (a * b)\angle(\theta_1 + \theta_2)$
- $a\angle\theta_1 / b\angle\theta_2 = (a / b)\angle(\theta_1 - \theta_2)$
- $6\angle 45^o / 2\angle 30^o = (6 / 2)\angle(45 - 30) = 3\angle 15^o$

Electrical Principals

Electric and Magnetic fields

- Introduced to E & M fields associated with antennas

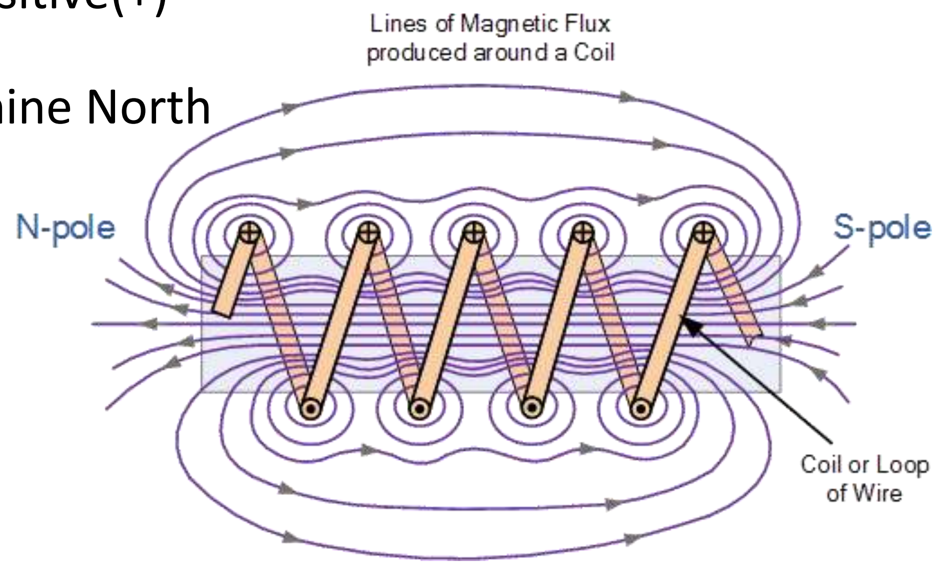


RC and RL Time Constants

- Electrical energy
  - Can be detected as voltage differences between two points
  - Capacitors store and release electrical energy
  - Capacitors will resist a change in voltage
- Magnetic energy
  - Is detected by moving electrical charges (current)
  - Inductors store and release magnetic energy
  - Inductors will resist a change in current movement

Current / Magnetic field direction

- Electronics (Elect Engr) use “whole” or “conventional” current
  - Current flows positive (+) → negative (-)
  - Use “Right Hand rule” to determine North
- Physics use “electron” current
  - Current flows negative (-)→ positive(+)
  - Use “Left Hand rule” to determine North



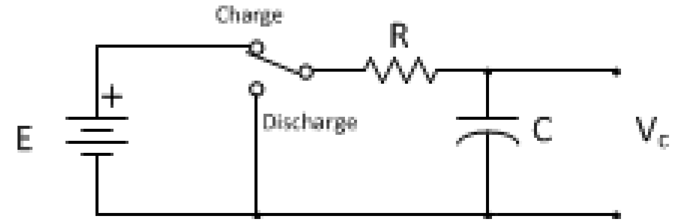
Time Constant

- RC circuit - The time it takes to charge (or discharge) a capacitor (V_C) depends on the size of the resistor

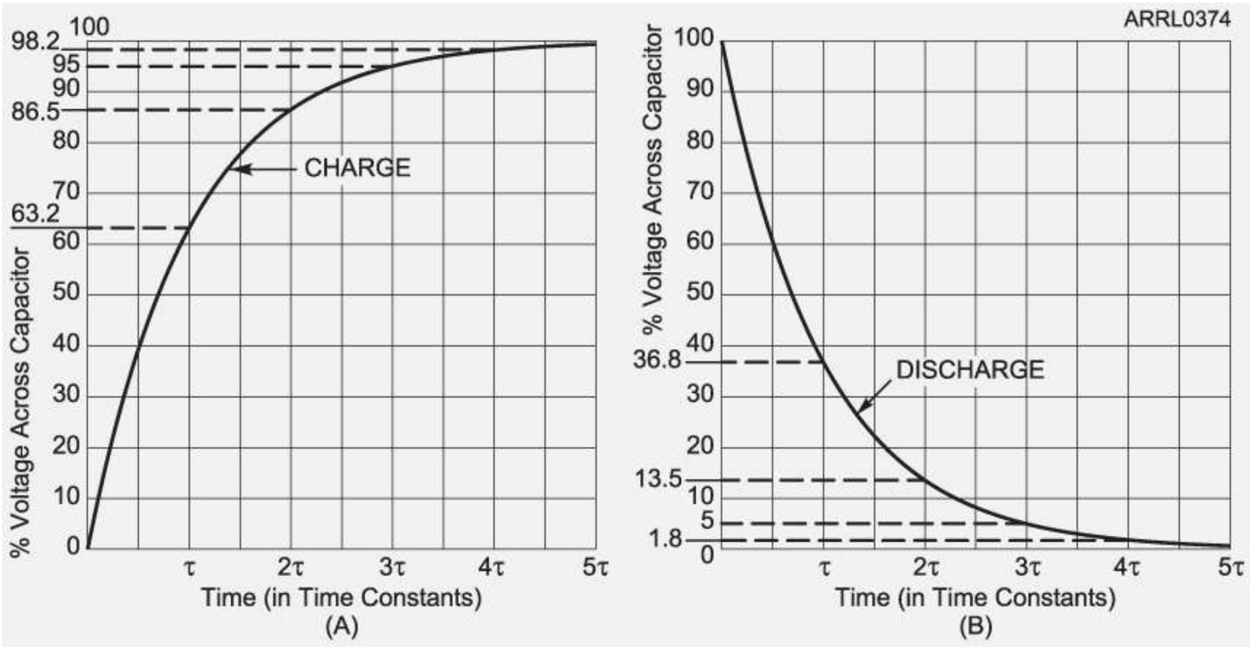
- RL circuit - The time it takes for current to increase (or decrease) through an inductor (i_L) also depends on the size of the resistor.

RC Circuit

- When charging a capacitor, $V_C\,(t) = E(1 - e^{-\frac{t}{\tau}})$ where τ = RC
- When discharging a capacitor, $V_C\,(t) = E(e^{-\frac{t}{\tau}})$
- e = 2.71828 (base for natural logarithms)

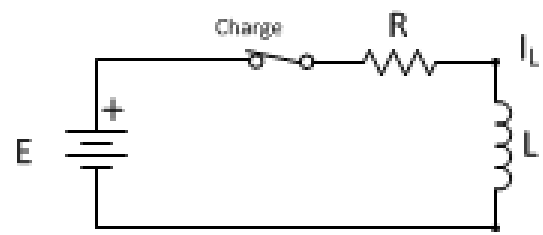


- It takes approximately 5 time constants to charge to 99.3% (or discharge to 0.7%)



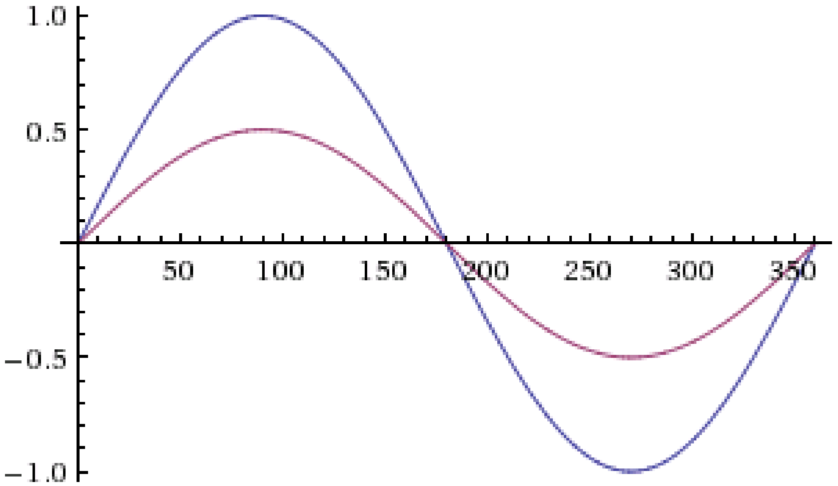
RL Circuit

- Building current through an inductor, I_L (t)= $\frac{E}{R} * (1 - e^{-\frac{t}{\tau}})$ where τ = L/R
- When disrupting current through an inductor, I_L (t)= $\frac{E}{R} * (e^{-\frac{t}{\tau}})$
- It takes approximately 5 time constants to charge to 99.3% (or discharge to 0.7%)



Phase Angle

- Phase Angle refers to time
  - 1 cycle of frequency x = 1/x seconds
  - 1 cycle = 360°
  - Phase angle is measured between similar points on each waveform
- Resistors dissipate energy
  - voltage & current are "in phase"

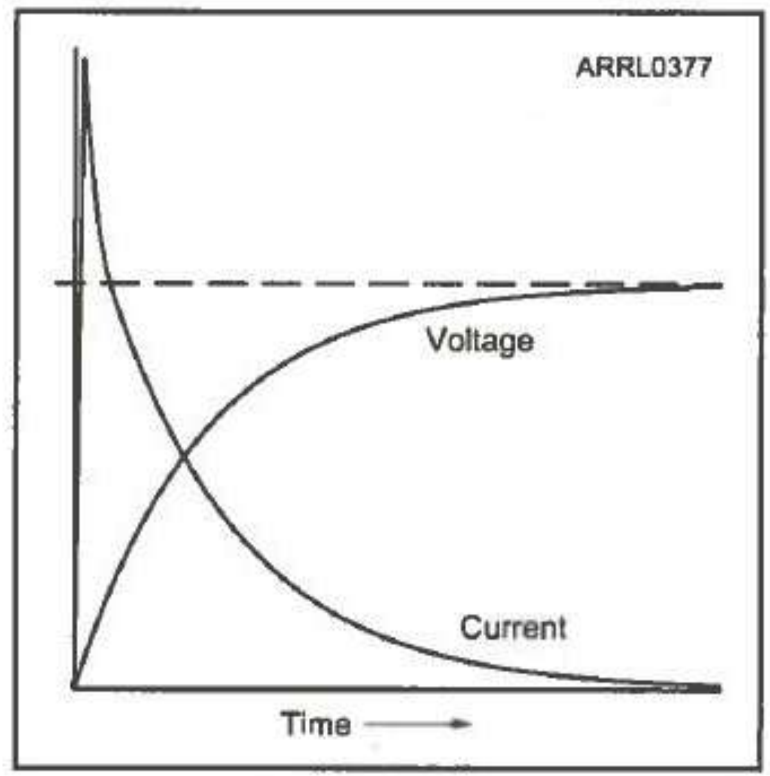


Phase Angle

- Capacitors and Inductors store / release energy
  – Unlike in Resistors, voltage & current waveforms do NOT rise and fall together
- ELI the ICE man
  – Inductors – voltage (E) leads current (I)
  – Capacitors – current (I) leads voltage (E)

Voltage/Current Relationship in Capacitors

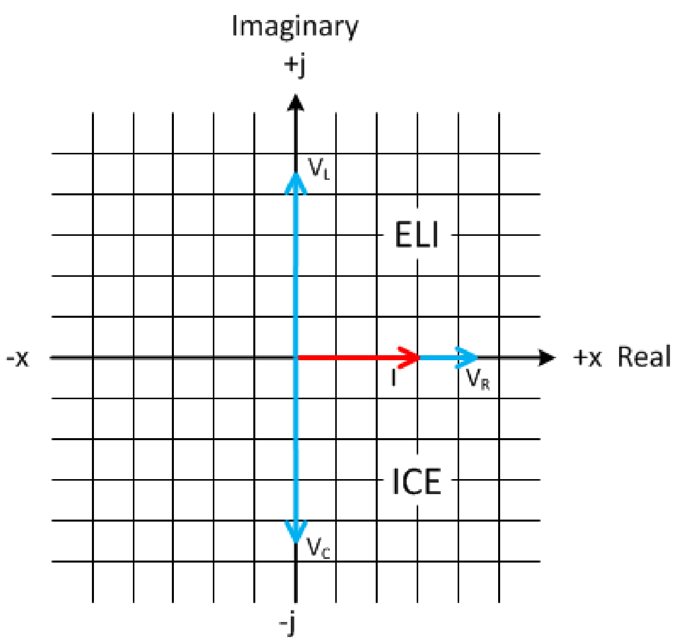
- Capacitors **resist** changes in voltage
- DC voltage / current
  - Physical example, filling a tank with air

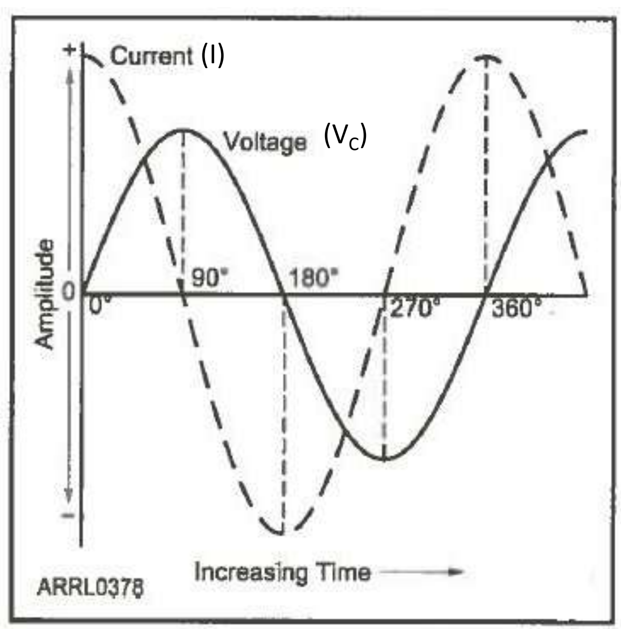


AC Voltage/Current phase relationship in Capacitors

ICE - current LEADS voltage (V_C)

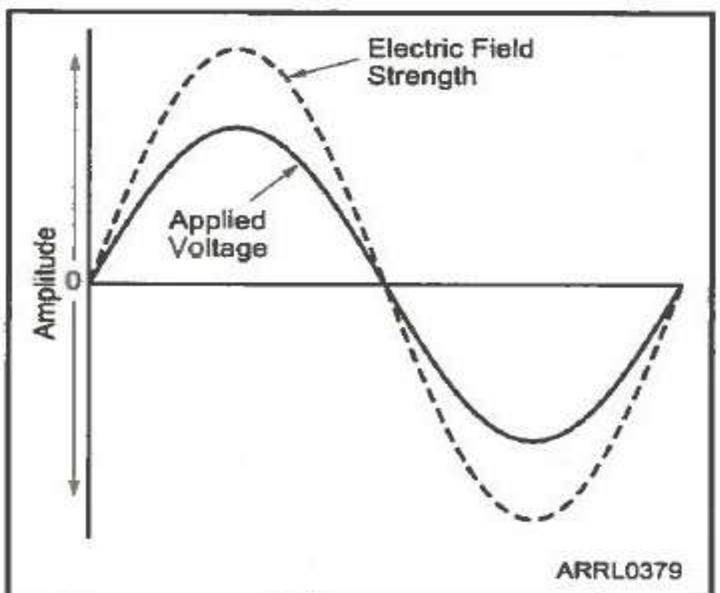
- voltage (V_C) **<u>LAGS</u>** current
- (current is reference) phase angle is -90º

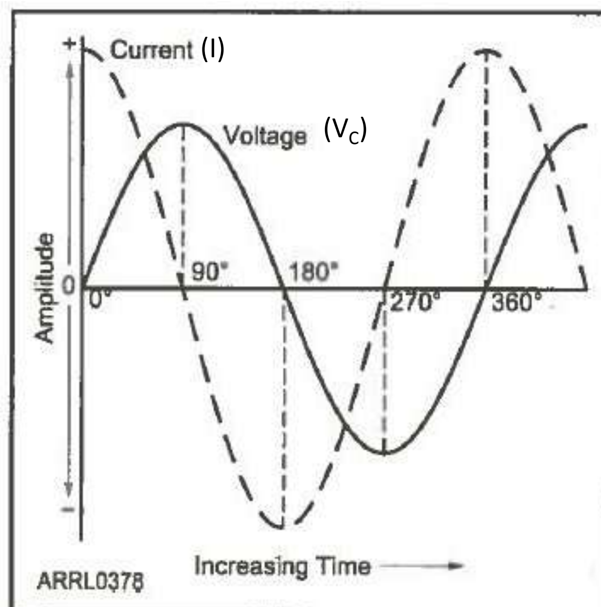




Electric (field strength) energy is stored and released twice each cycle

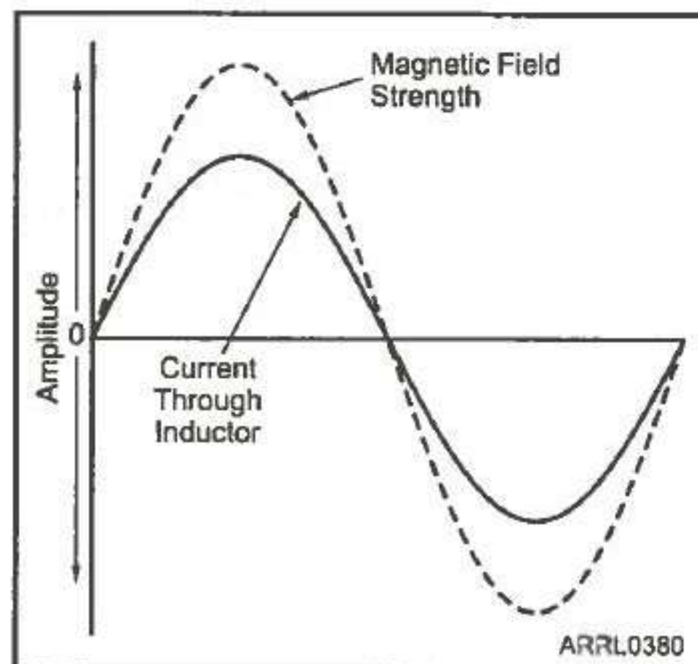
- Max current at 0 volts
- 0 current at Maximum volts
- Current leads voltage (ICE)





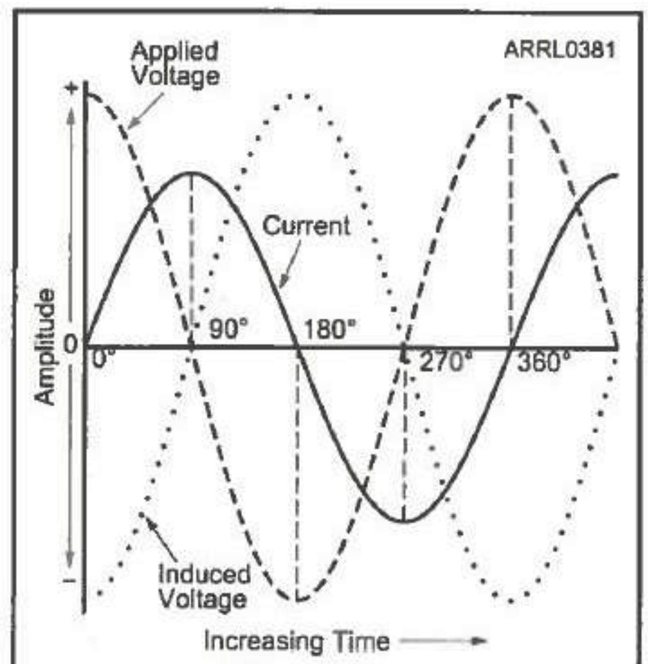
Voltage/Current Relationship in Inductors

- Inductors **<u>resist</u>** changes in current
  - Physical example, flywheel
- Relationship between AC voltage and current (in an inductor) complements that in a capacitor
  - Stored Magnetic (field) energy is in phase with current



Back EMF (induced voltage) in Inductors

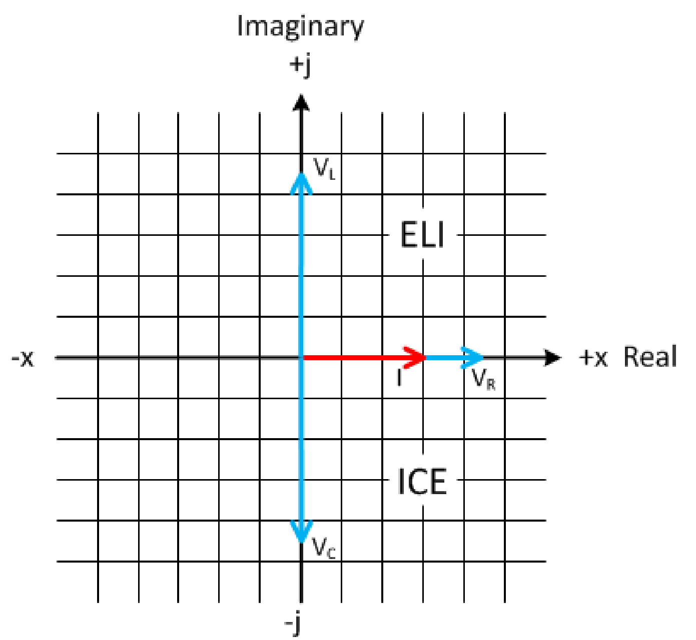
- is greatest when the magnetic field is changing the fastest
- is generated with a polarity that opposes the change in current



AC Voltage/Current phase relationship in Inductors

ELI - current LAGS voltage (V_L)

- voltage(V_L) <u>**LEADS**</u> current
- (current is reference) phase angle is $+90^o$



Resistance versus Reactance

- Resistance is the opposition to the passage of (DC or AC) current (ohm Ω)
- Reactance is the opposition to the AC current flow through an inductor or capacitance
  - Inductive reactance
  - Capacitive reactance

Inductive Reactance

- Increases with <u>increasing</u> frequency
- $X_L = 2\pi f L$

where

X_L =reactance in ohms

f = frequency in hertz

L = inductance in henries

Capacitive Reactance

- Increases with <u>decreasing</u> frequency
- $X_C = 1/2\pi f C$

where

X_C =reactance in ohms

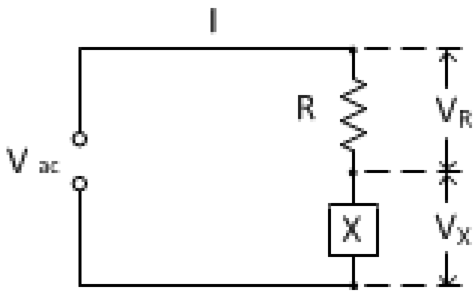
f = frequency in hertz

C = capacitance in farads

Complex Impedance

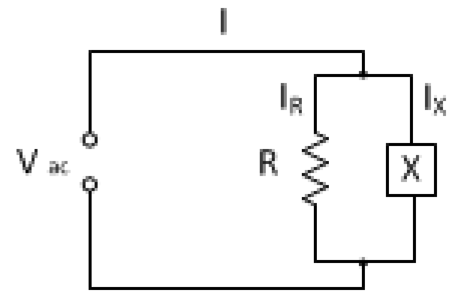
- Impedance (Z) is composed of two components, resistance and reactance
- Reactance can be either inductive or capacitive
- Resistance and Reactance may be connected either in series or parallel

Series Circuit



- Current (I) is the same through both elements (R & X)
  - $I = I_R = I_X$
- Voltages (across the elements) are different (V_R & V_X)
  - $V_{ac} = V_R + V_X$

Parallel Circuit



- Voltage (V_{ac}) is the same across both elements (R & X)
  - $V_{ac} = V_R = V_X$
- Currents (through each element) are different (I_R & I_X)
  - $I = I_R + I_X$

Complex Impedance

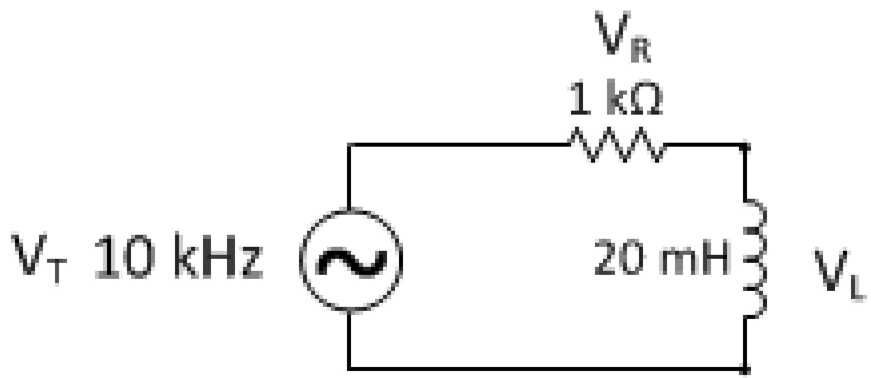
The phase relationship between the current (I) and voltage (for the whole circuit, V_{ac}) can be between 0^o and $\pm 90^o$

- The phase angle depends on the relative amounts of resistance and reactance
- The sign of the angle depends on the type of the reactive element
  - Inductive reactance produces a positive angle
  - Capacitive reactance produces a negative angle

Reduce multiple elements to one equivalent element

- Multiple resistors to one resistor
- Multiple reactive elements (capacitors / inductors) to one reactive element
  - Capacitors (-90^o) and inductors ($+90^o$) have opposite phase angles
  - Combining capacitors and inductors will result in a smaller TOTAL reactance (adding the negative and positive angles will cancel each other)
  - If the capacitors and inductors have EQUIVALENT reactive values, they will cancel each other (angle=0^o thus X=0), resulting in only a resistive element (R)

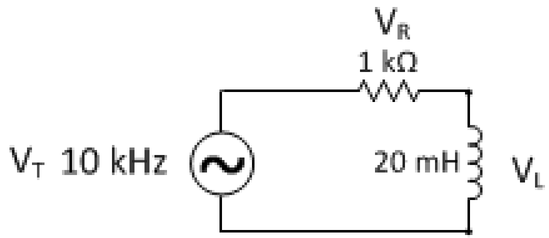
Calculate Impedance



Calculate Reactance of 20 mH inductor at 10 kHz

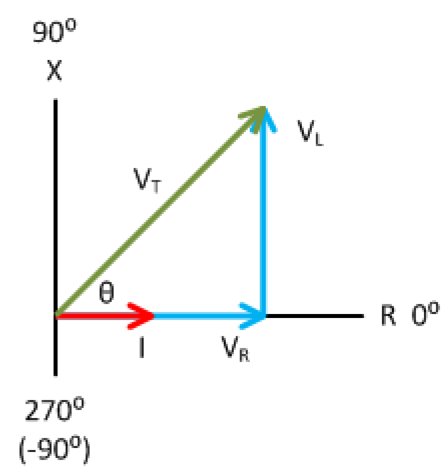
- $X_L = 2\pi f L$
- $X_L = 2 \times 3.14 \times (10 \times 10^3) \times (20 \times 10^{-3})$
- $X_L = 1256\Omega$

Calculate Impedance

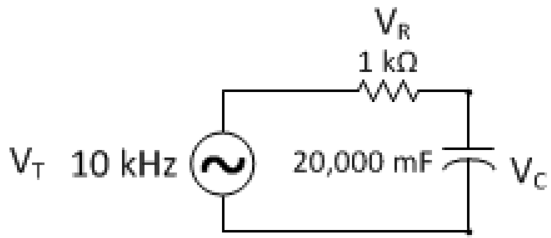


Since we aren't given a V_T, we assume I=1

- V_R = 1000v (voltage in phase with current)
- V_L = 1256v (voltage leads current by 90°, "ELI")
- $V_T = \sqrt{1000^2 + 1256^2}$ = 1605
- $\theta = \tan^{-1}\left(\frac{opp}{adj}\right) = \tan^{-1}\left(\frac{1256}{1000}\right) = 51.5°$
- V_T = 1605∠ 51.5°
- $Z = \frac{V_T}{I} = \frac{1605\angle 51.5^0}{1\angle 0^0} = 1605\angle 51.5^0\ \Omega$

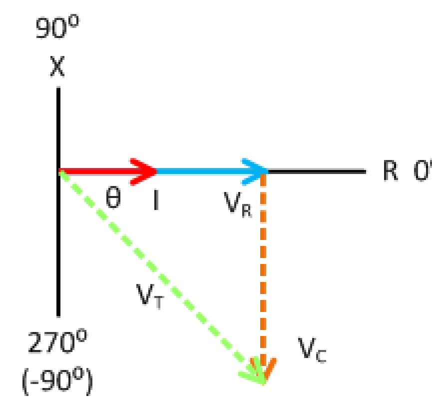


Calculate Impedance



What if we would have had a 20,000 mF capacitor?

- $X_C = 1/(2\pi\ f\ C)$
- $X_C = 1/[2 \times 3.14 \times (10 \times 10^3) \times (20{,}000 \times 10^{-6})]$
- $X_C = 1/1256\Omega = 1256\angle -90^o$
- voltage lags current by 90^o
- OR current leads voltage by 90^o, "ICE"
- $Z = 1605\angle -51.5^0\ \Omega$



Calculating Impedances and Phase Angles - BASIC Rules

- Impedances <u>in series</u> add together
- Admittance is the reciprocal of impedance

  Y = $\frac{1}{Z}$

- Admittances <u>in parallel</u> add together
- Inductive and capacitive reactance in series cancel
- $\frac{1}{j} = -j$

Refresher

- Conductivity (G) is the reciprocal of Resistance (R)

  G = $\frac{1}{R}$ units of siemens (S)

- Susceptance (B) is the reciprocal of Reactance (Y)

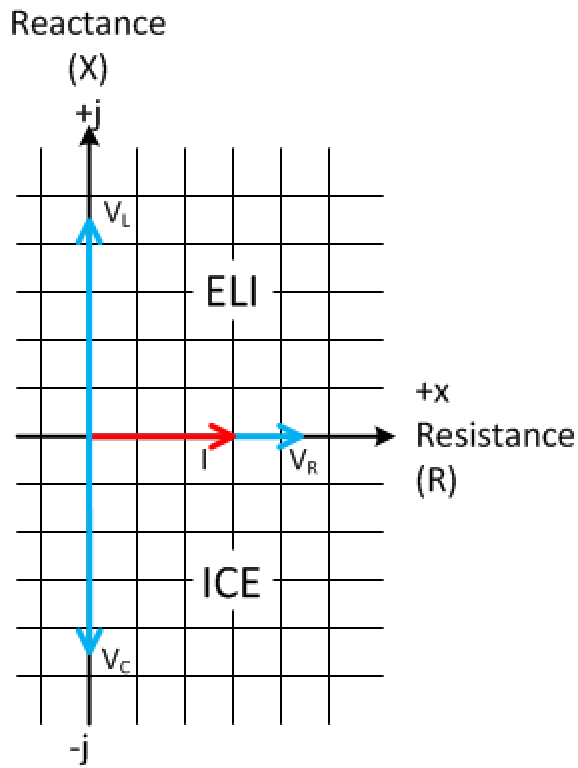
  B = $\frac{1}{Y}$ units of siemens (S)

- Taking the reciprocal of an angle, changes its sign

  $37^o = \frac{1}{-37^0}$ and $-73^o = \frac{1}{73^0}$

Refresher (cont)

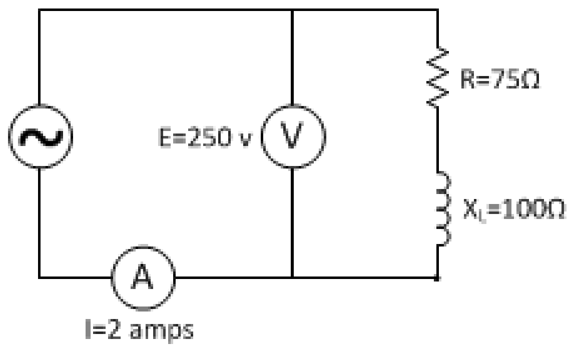
- Voltage determines the reference for phase angle
  - Current is the reference
  - If voltage LEADS current (ELI) -> Positive phase angle
  - If voltage LAGS current (ICE) -> Negative phase angle
    - Voltage LAGS current = current leads voltage



Reactive Power and Power Factor

- Power = Voltage x Current (DC circuit)
- Reactive Power = Voltage_{RMS} x Current_{RMS}
  – AC circuit, when voltage & current are in phase
- Apparent Power
  – Does not take into consideration phase angle
  – Expressed in units of volt-amperes (VA)
  – AKA reactive, nonproductive or wattless power (VAR)

Power Factor

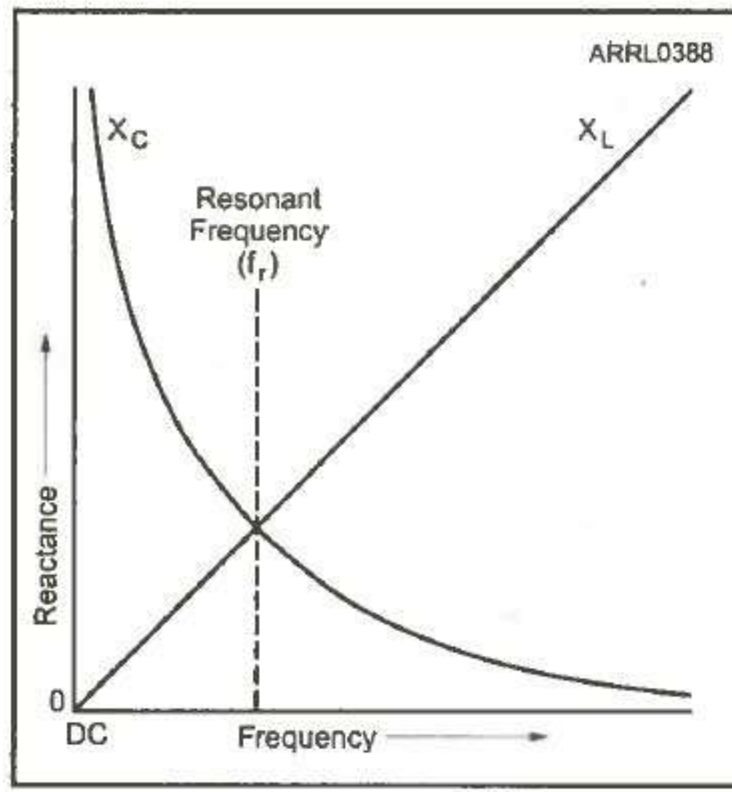


$$\mathrm{PF} = \frac{P_{REAL}}{P_{APPARENT}}$$

- Apparent power = 250 V x 2 A = 500 VA
- Real power = I^2 R = $(2\ \mathrm{A})^2$ x 75 Ω = 300 W
- PF = $\frac{Real}{Apparent}$ = $\frac{300\ W}{500\ VA}$ = 0.6
- Power Factor is calculated using phase angle (θ) between the current and voltage [PF= Cos (θ)]
  - PF = 1 when θ = 0^o (all apparent power is real power)
  - PF = 0 when θ = 90^o (all apparent power is reactive power)

Resonant Circuits

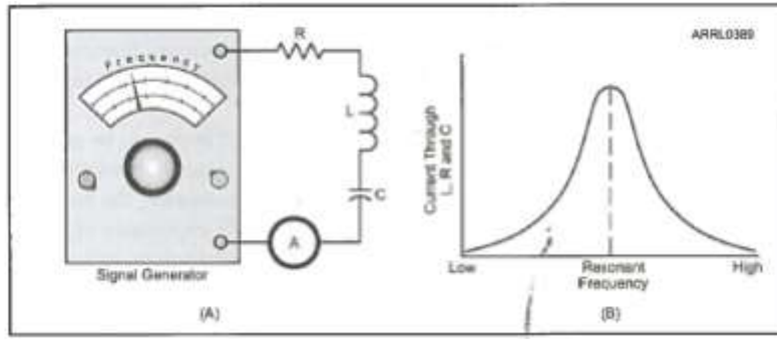
- Circuits with different inductive and capacitive reactances
  - Series – different voltages
  - Parallel – different currents
- Circuit is said to be resonant when $X_L = X_C$
  - Series – V_C will cancel V_l, leaving only V_R
  - Parallel - I_C will cancel I_l, leaving only I_R
- Calculation of Resonant frequency
  - $X_L = X_C$
  - $2\pi f L = 1/2\pi f C$
  - Resonant frequency $(f_r) = \frac{1}{2\pi\sqrt{LC}}$



Impedance of Resonant **Series** Circuits

Series Circuits

- Same current
- V_L is 180^o out of phase with V_C

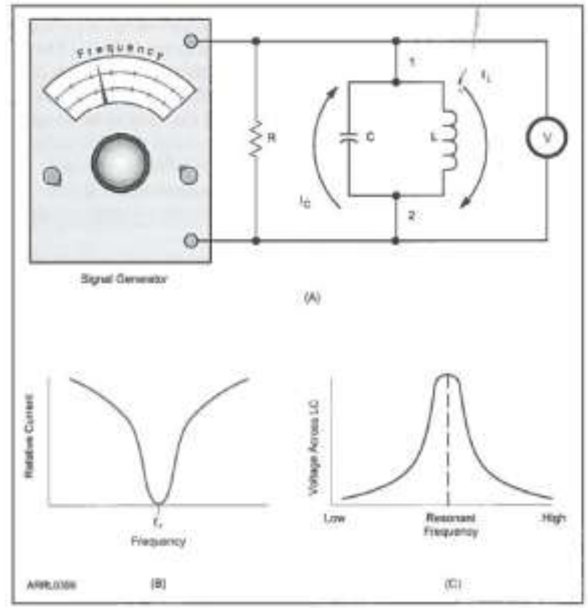


- At resonant
  - $V_L = V_C$
  - Equal amounts of energy are stored in each component
  - The energy being supplied will cause voltages across the inductor and capacitor to build to multiple times the source voltage
  - X_L cancels X_C, resulting in maximum current
  - Voltage and current are in phase

Impedance of Resonant **Parallel** Circuits

Parallel Circuits

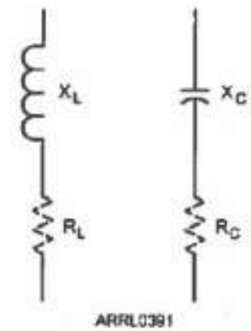
- Same voltage
- I_C is 180^o out of phase with I_L



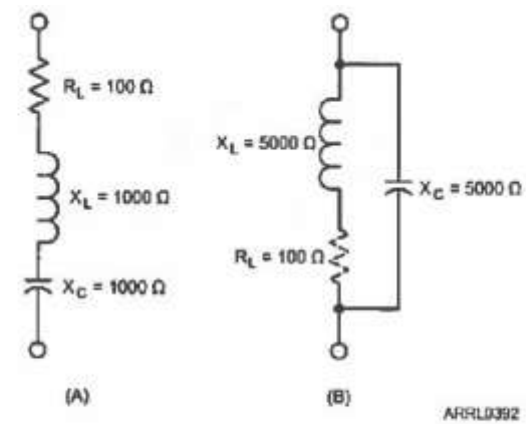
- At resonant
  - $I_L = I_C$
  - $I = (I_L + I_C) = 0$, so the parallel combination will appear like an open circuit
  - The energy being exchanged between the inductor and capacitor will result in large circulating current
  - Total current from the generator is small
  - Voltage (across the tank) is at maximum
  - Voltage and current are in phase

Q and Bandwidth of Resonant Circuits

- Practical components
  - Can be represented as ideal components (inductor or capacitor) in series with a resistor
  - That resistance dissipates some of the stored energy



- Quality factor (Q)
  - Represents how close to an ideal component is the practical component
  - $Q = \frac{X}{R}$
  - Ratio of how much energy is stored to how much energy is dissipated
  - Energy losses of a capacitor is usually much less than that for an inductor
  - Thus the Q of the inductor is usually the limiting factor of the Q of a resonant circuit
  - The only way to raise the Q of an inductor or capacitor is by using components with less internal resistance



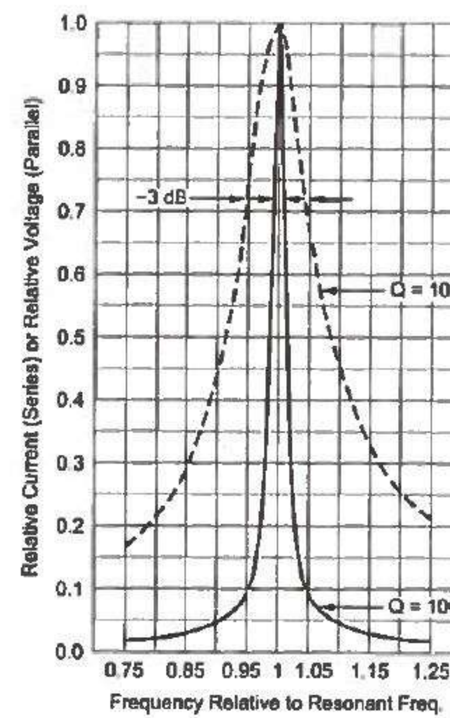
Q and Bandwidth of Resonant Circuits (cont)

- Resonant Circuit Bandwidth
  - Bandwidth of the frequency range where the voltage (or current) is no more than 3 dB (i.e. .707) below the peak
  - Also called the half-power (or -3 dB) bandwidth
  - $\Delta f = \frac{f_r}{Q}$

    $\Delta f =$ the half power bandwidth

    $f_r =$ the resonant frequency of the circuit

    $Q =$ the circuit Q
  - Higher Q, the narrower the bandwidth (sharper)



Q and Bandwidth of Resonant Circuits (cont)

- Skin Effect and Q
  - As frequency increases
    - more of the current travels closer to the surface of the wire
    - the “effective” resistance of the wire increases
  - HF – outer few thousandths of an inch
  - VHF & UHF – outer few ten-thousandths

Magnetic Cores

- Inductors store magnetic energy, creating reactance
- Wire can be wound around various material
  - Air is relatively inefficient way to store magnetic energy
  - Magnetic material increases the storage of energy (and the inductance)
- Inductance is a function of
  - Number of turns of wire on the core
  - Core materials permeability (air = 1)

Magnetic Cores (cont)

- Select the core material carefully
  - Perform over a desired frequency range
  - Temperature stability
- Core shape
  - Affects magnetic field (shape around the inductor)
  - Magnetic field of one inductor can interact with nearby components (coupling)
  - Toroid cores reduce unwanted coupling by keeping the magnetic field contained in the “donut”

Magnetic Cores (cont)

- Calculating Inductance
  - $\mathrm{L} = \frac{A_L N^2}{10{,}000}$
  - Or number of turns $N = 100\sqrt{\frac{L}{A_L}}$
  - Each time the wire passes through the core, it counts as a turn
- Ferrite bead
  - Small core, slipped over component leads, often used as suppressors for VHF & UHF oscillations

Questions